NBX-003-016404 Seat No. _______

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2017

Mathematics : CMT - 4004

(Graph Theory)

Faculty Code : 003

Subject Code : 016404

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Each question carries **equal** marks.

(2) All the questions are **compulsory**.

1 Answer the following short questions : **7×2=14**

(i) Define walk, trail, path and cycle.

(ii) Define adjacency matrix $X(G)$ for a graph G.

(iii) Define isomorphism of two graphs.

(iv) Give example of two graphs G_1, G_2 which satisfy

(i) $|V(G_1)| = |V(G_2)|$, (ii) $|E(G_1)| = |E(G_2)|$ and (iii) for any integer $m \geq 0$, the number of vertices in G_1 with degree m is same as the number of vertices in G_2 with degree m, however G_1 and G_2 are not isomorphic graphs.

(v) Write down all the spanning trees for following graph G :

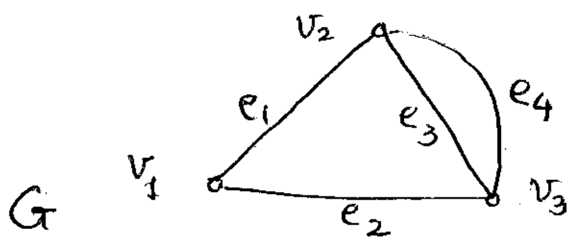

Fig. 1

(vi) Give definitions of k-regular graph and simple graph. Draw a 3-regular simple graph.

(vii) Give statement of Euler's theorem and draw a graph which is Eulerian graph and its all vertices have not same degree.

2 Attempt any **two** : **2×7=14**

(a) Let G be a simple graph with n vertices, q edges and k components. In standard notation prove that

$$q \leq \frac{1}{2}(n-k)(n-k+1).$$

(b) Let G be a connected graph and $d_G(v) = even, \forall v \in V(G)$. Prove that G must be an Eulerian graph.

(c) Let u and v be two vertices of a tree $T(u \neq v)$. Prove that there is a unique path P between u and v in T.

3 Attempt any **one** : **1×14=14**

(a) Let $G = (V, E)$ be a connected graph. Prove that the ring sum of two cut sets of G is either a cutset for G or it is an edge disjoint union of two cut sets of G.

(b) Let G be a simple graph with n vertices and $n \geq 3$. Let G satisfies $d_G(v) \geq \frac{n}{2}. \forall v \in V(G)$. Prove that G is a Hamiltonian graph. Also define Hamiltonian graph and maximal non-Hamiltonian graph.

(c) Prove Max-flow min-cut theorem : In a given transport network G, the maximum value of a flow from s to t is equal to the minimum value of the capacities of all the cuts in G that separate s from t.

4 Attempt any **two** : **2×7=14**

(a) Let $G = (V, E)$ be a graph which does not contains any loop. Also every pair $u, v \in V$, there is a unique path between u and v in G. Prove that G must be a tree.

(b) Let T be a tree on n vertices. Prove that T has either one or two centers. Also deduce that in the case T has two centers, they must be adjacent by an edge in T.

(c) Define separable graph and cut-vertex. For a separable graph $G=(V, E)$, prove that $v \in V$ is a cut-vertex for G if and only if there is atleast two vertices $x, y \in V-\{v\}$ such that every path between x and y in G passes through v.

(d) Let G be a connected plannar graph with n vertices, e edges and f regions (faces). In standard notation derive the formula $f = e - n + 2$.

5 Attempt any **two** : **2×7=14**

(a) Let T be a tree with $|V(T)| \geq 2$. Prove that T is a 2-chromaticgraph.

(b) Find out the adjacency matrix X for following graph G. Also find the matrix $Y = X + X^2 + X^3 + X^4$. Is G connected ?

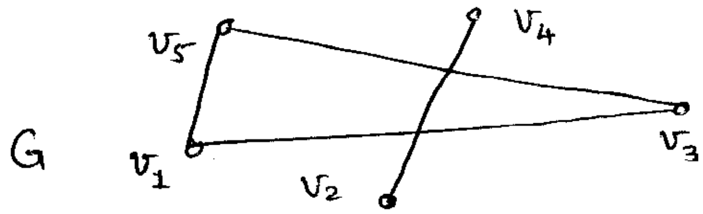

Fig. 2

(c) Prove that Kuratowaski's first graph K_5 and second graph $K_{3, 3}$ both are not plannar graphs.

(d) Let G be a connected graph, S be a cut set in G and F be a cycle in G. In standard notation prove that $|E(F) \cap S|$ is even.
